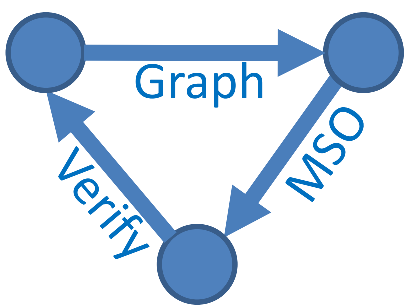
Graph Query Verification using Monadic 2nd-Order Logic

Kazuhiro Inaba (稲葉 一浩)

kinaba@NII.ac.jp



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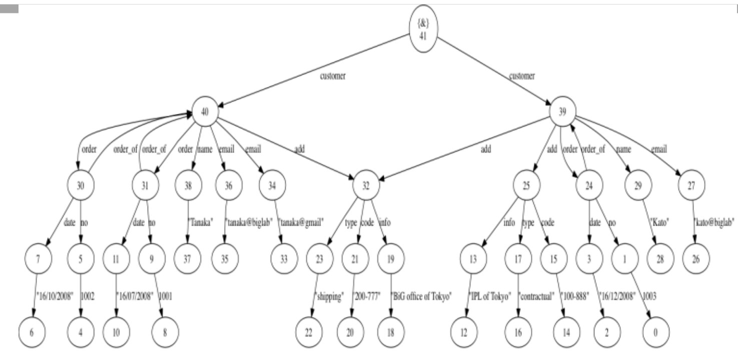
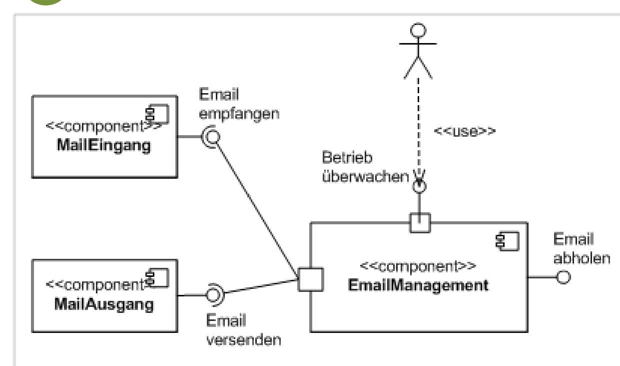
Goal of This Research

(Automated) Reasoning
on Graph Transformations



Is this update on the output reflectable to the input?

Does the output of this transformation always have a desired structure?

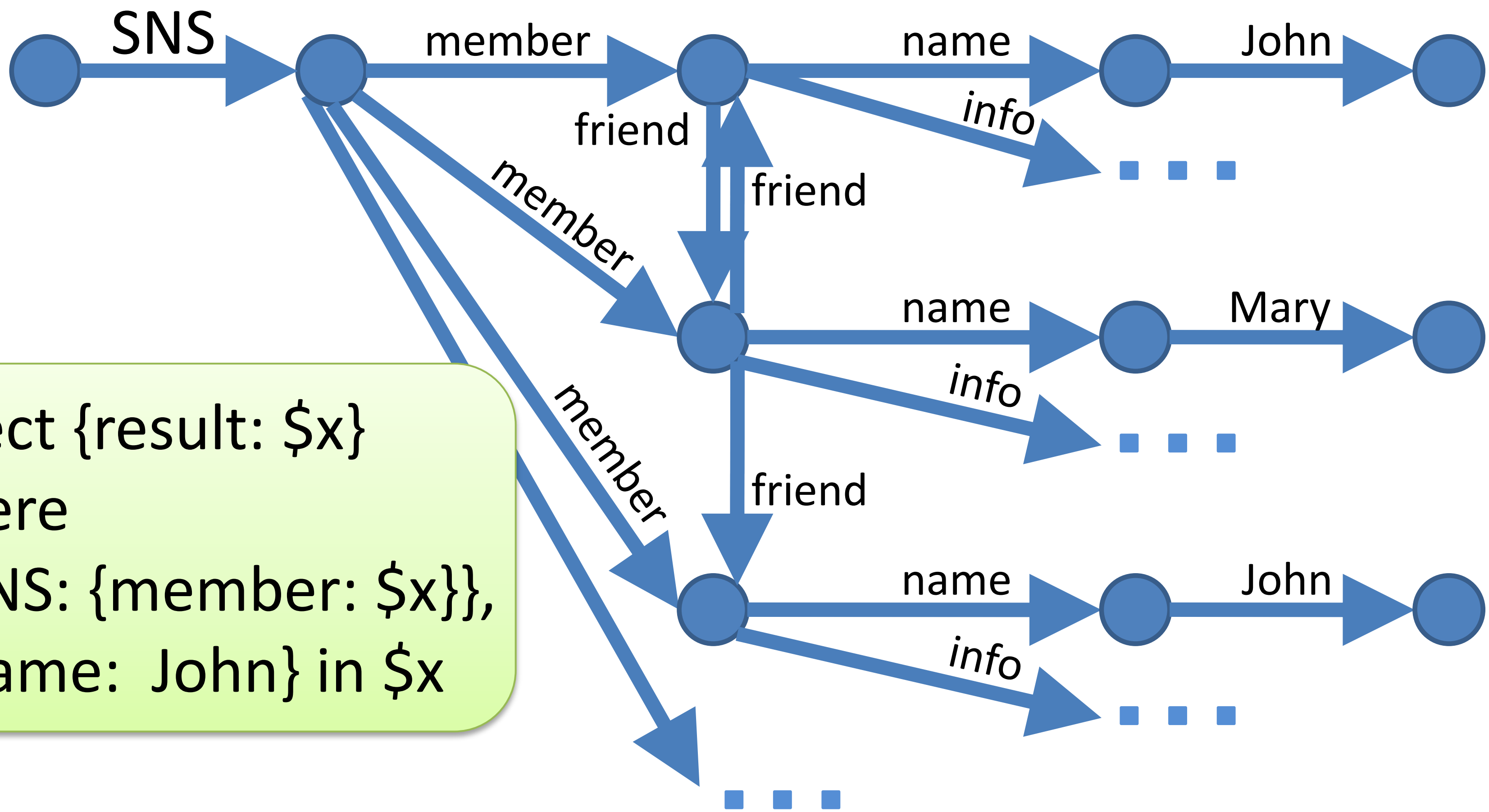


Today's Talk

- Given
  - A graph transformation f
  - Input schema S_I
  - Output schema S_O
- Statically verify that "there's no type error", i.e., **"for any graph g conforming to S_I, $f(g)$ always conforms to S_O."**

Example : SNS-Members

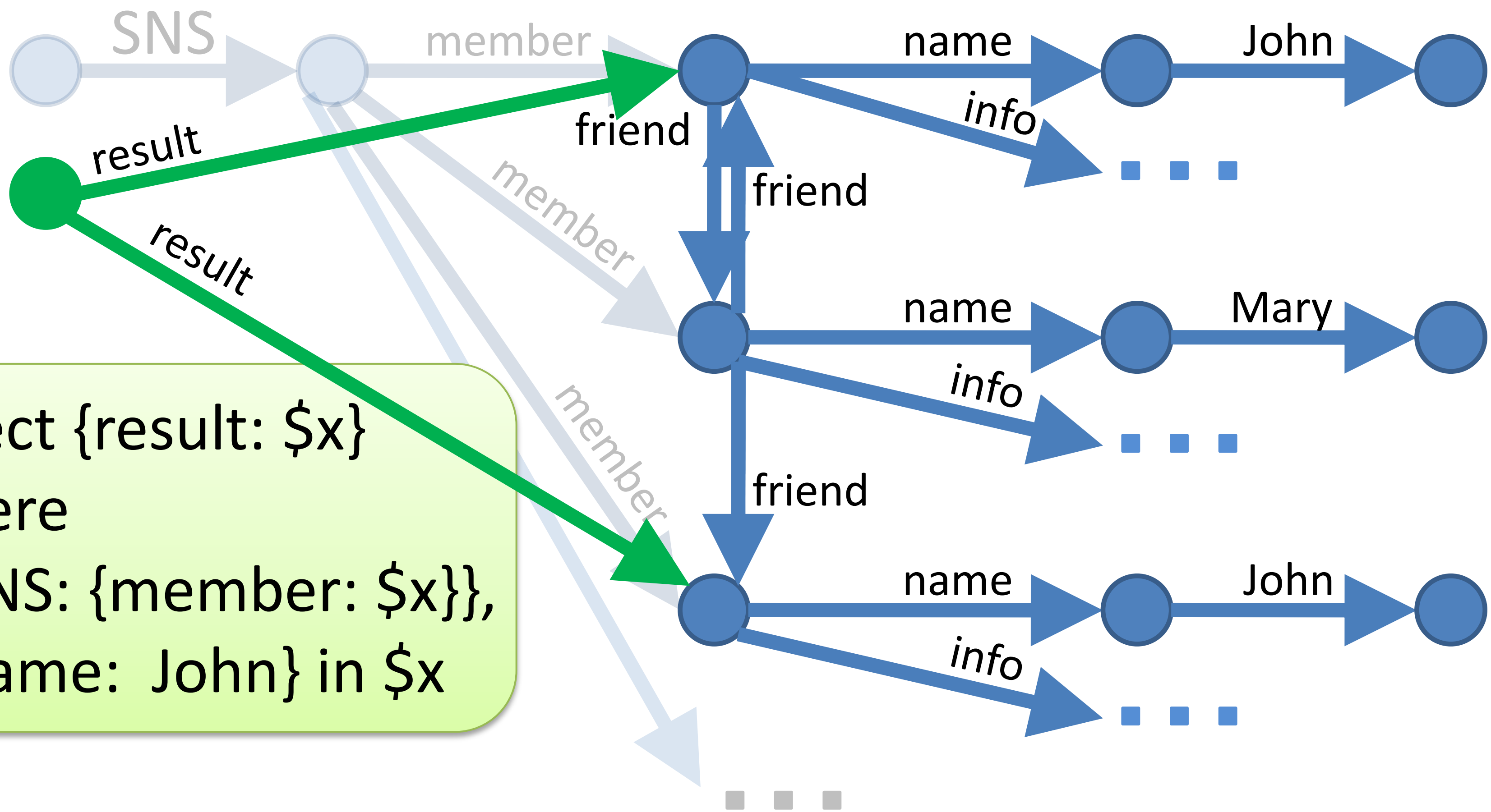
Extract all members using the screen-name “John”.




```
select {result: $x}
where
  {SNS: {member: $x}},
  {name:  John} in $x
```

Example

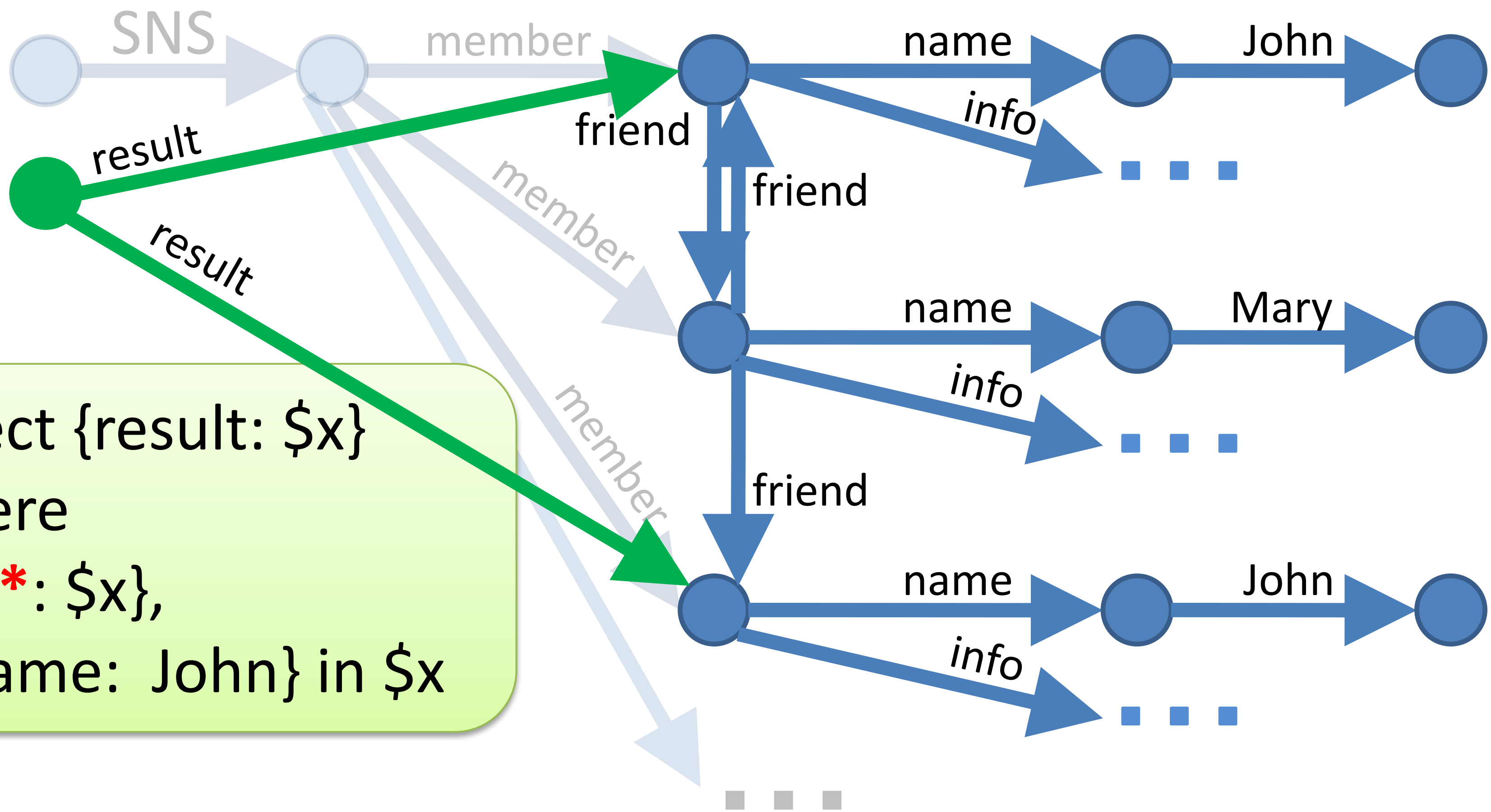
Extract all members using the screen-name “John”.




```
select {result: $x}
where
  {SNS: {member: $x}},
  {name:  John} in $x
```

Example

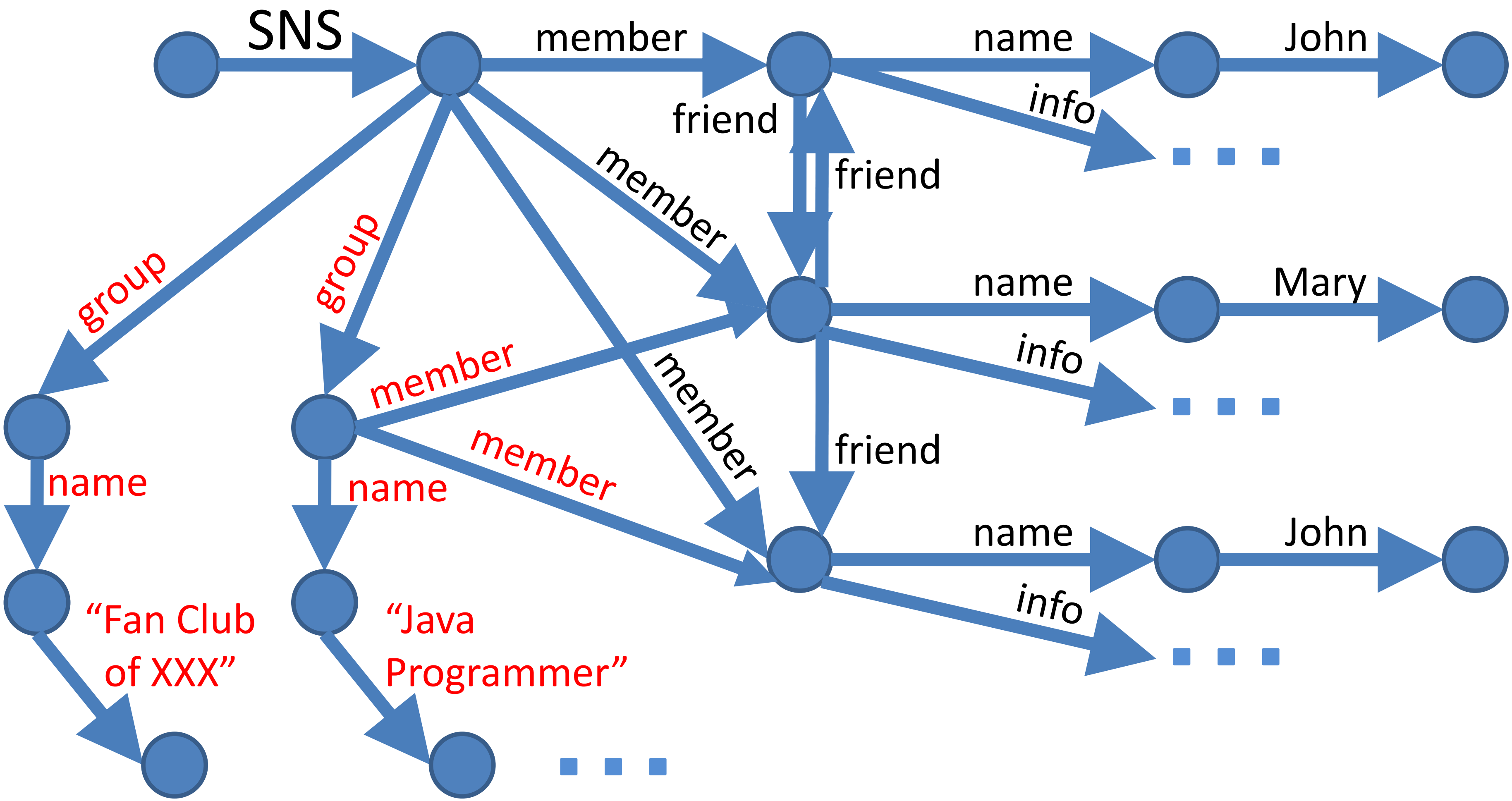
Lazy programmer may write …



select {result: $x}
where
{ _*: $x},
{name: John} in $x

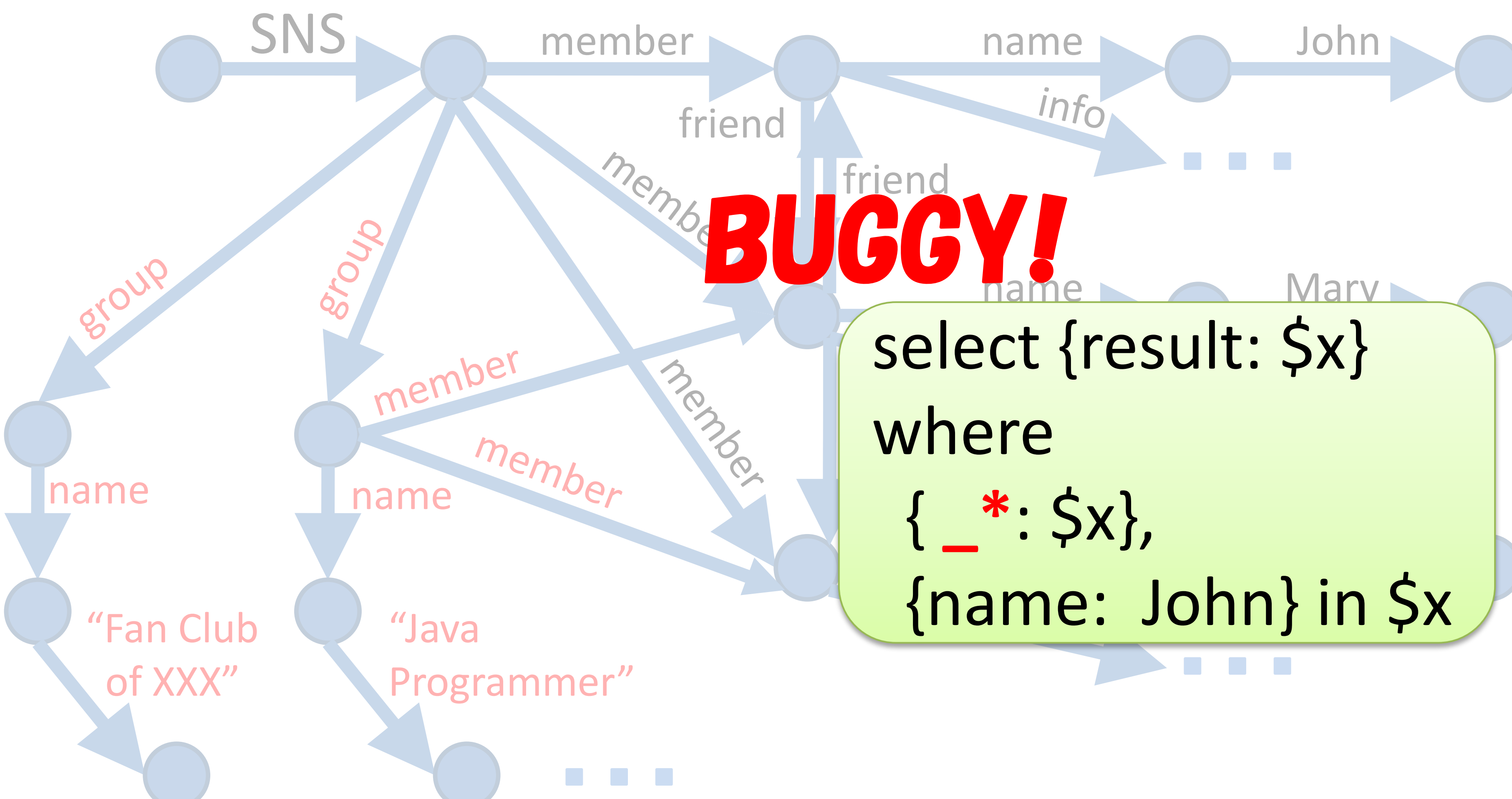
Example

In fact, the graph contained “group” data, too!



Example

What happens if there's {group: {**name: John**, …}}




```
select {result: $x}
where
  {_*: $x},
  {name:  John} in $x
```

What We Provide

Programmers specify their intention about the structure of input/output.

class OUTPUT { reference result*: MEM; }

```
// Input Schema supplied by the SNS provider
class INPUT  { reference SNS: SNSDB; }
class SNSDB  { reference member*: MEM;
               reference group*: GRP; }
class MEM    { reference friend*: MEM;
               reference name: STRING; }
class GRP    { reference name: STRING;
               reference member*: MEM; }
```

What We Provide

Then, our system automatically verify it!

```
class INPUT {
 reference SNS: SNSDB; }
```

```
select {result: $x}
where
  {SNS: {member: $x}},
  {name:  John} in $x
```

```
class OUTPUT {
 reference result*: MEM; }
```

⇨ **"OK!"**

※ Our checker is SOUND.
If it says OK, then the program never goes wrong.

What We Provide

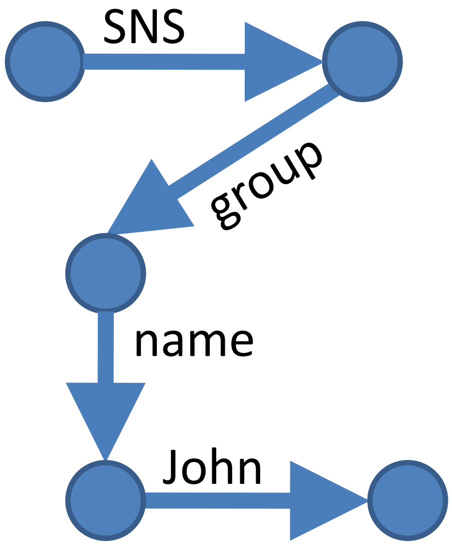
Then, our system automatically verify it!

```
class INPUT {
 reference SNS: SNSDB; }
```

```
select {result: $x}
where
  {_*: $x},
  {name: John} in $x
```

```
class OUTPUT {
 reference result*: MEM; }
```

"BUG!"



※ Our checker provides a COUNTER-EXAMPLE.

How?

By encoding transformations into a logic formula.

```
class INPUT {
 reference SNS: SNSDB; }
```

```
select {result: $x}
where
  {_*: $x},
  {name:  John} in $x
```

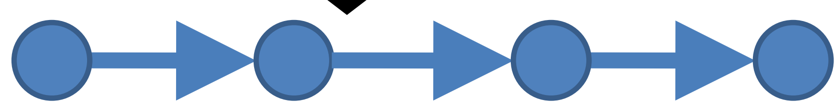
```
class OUTPUT {
 reference result*: MEM; }
```

formula

Check **VALIDNESS** using an existing verifier

"YES" / **"NO"+CE**

Decode CE to a graph



The Challenge

"How should we represent schemas and transformations by logic formulas?"

- The logic must not be too strong
  (otherwise its validness becomes undecidable)
- The logic must not be too weak
  (otherwise it cannot talk about our schemas and transformations)
- **What is the "just-fit" logic?**

Rest of the Talk

- Our Choice
  - **Monadic 2nd-Order Logic (MSO)**
- Schema Language
  - How it can be represented in MSO
- UnCAL Transformation Language
  - How, in MSO
- Decide MSO: from Graph-MSO to Tree-MSO
- Discussion : Why This Approach

Monadic 2^{nd}-Order Logic

MSO is a usual 1^{st} order logic on graphs …

(primitives) $edge_{foo}(x, e, y)$ $start(x)$

(connectives) $\neg P$ $P\&Q$ $P \vee Q$ $\forall x.P(x)$ $\exists x.P(x)$

… extended with

(set-quantifiers) $\forall^{set} S.\ P(S)$ $\exists^{set} S.P(S)$

(set-primitives) $x \in S$ $S \subseteq T$

Graph Schema Language

- Programmers can specify any MSO-expressible property
  (as long as it is bisimulation-generic & compact)
- For example,
  (count-free subset of) KM3 MetaModeling Language:

class OUTPUT { reference result*: MEM; }
class MEM { reference friend*: MEM;
reference name$^+$: STRING; }

Schema to MSO

- We do need MSO's expressiveness

class OUTPUT { reference result*: MEM; }
class MEM { reference friend*: MEM;
reference name: STRING; }

$$
\begin{aligned}
&\exists^{set}\mathrm{OUTPUT}.\ \ \exists^{set}\mathrm{MEM}.\\
&\quad (\forall x.\ \mathrm{start}(x) \rightarrow x \in \mathrm{OUTPUT})\\
&\wedge (\forall x \in \mathrm{OUTPUT}.\ \forall e.\ \forall u.\\
&\qquad \mathrm{edge}(x,e,u) \rightarrow \mathrm{edge}_{\mathrm{result}}(x,e,u)\ \&\ u \in \mathrm{MEM})\\
&\wedge \ldots
\end{aligned}
$$

Transformation Language

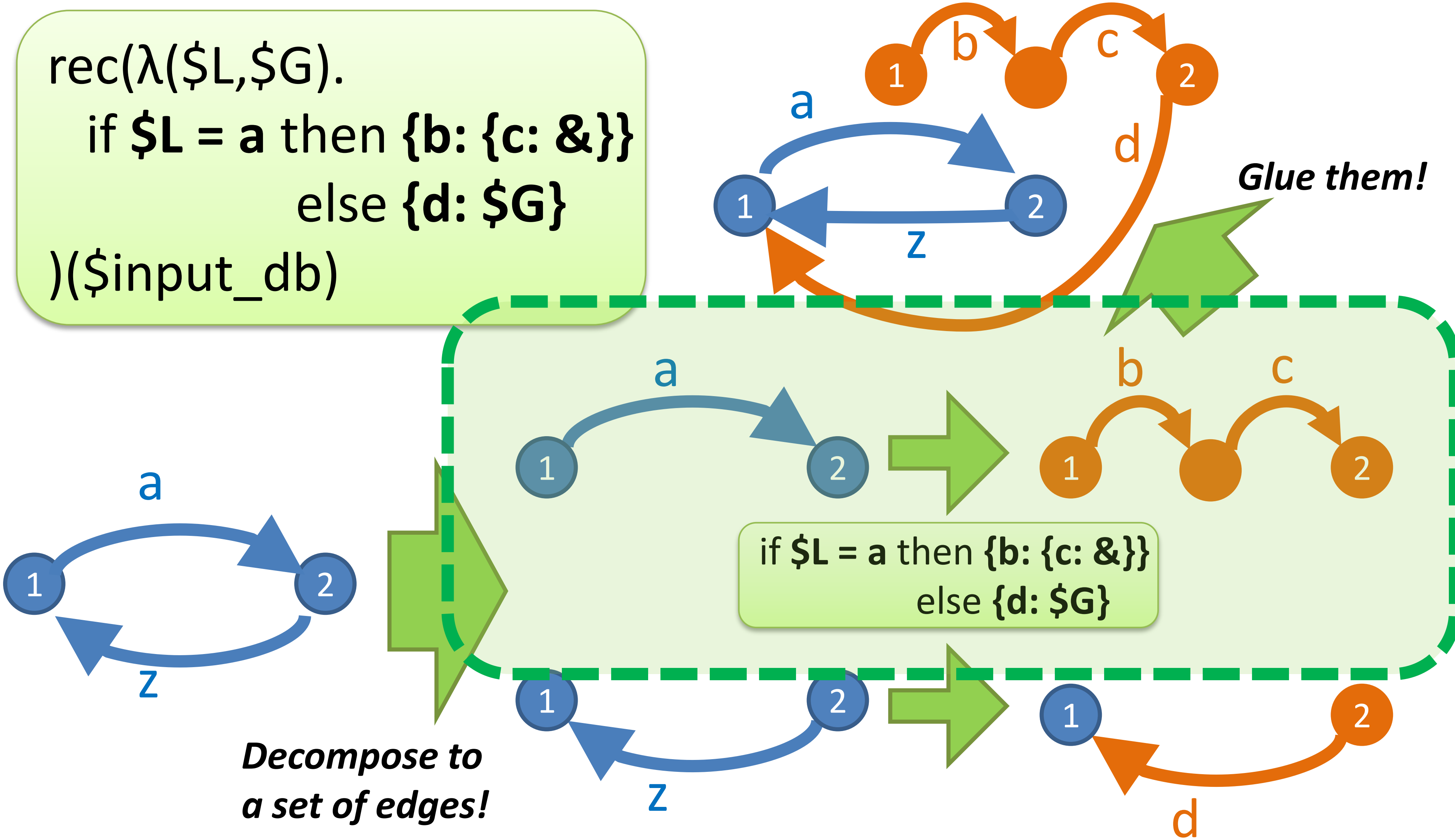
- UnCAL [Buneman et al, 2000]
  - Internal Representation of “UnQL” ➔

```
select {result: $x}
where
  {_*: $x},
  {name:  John} in $x
```

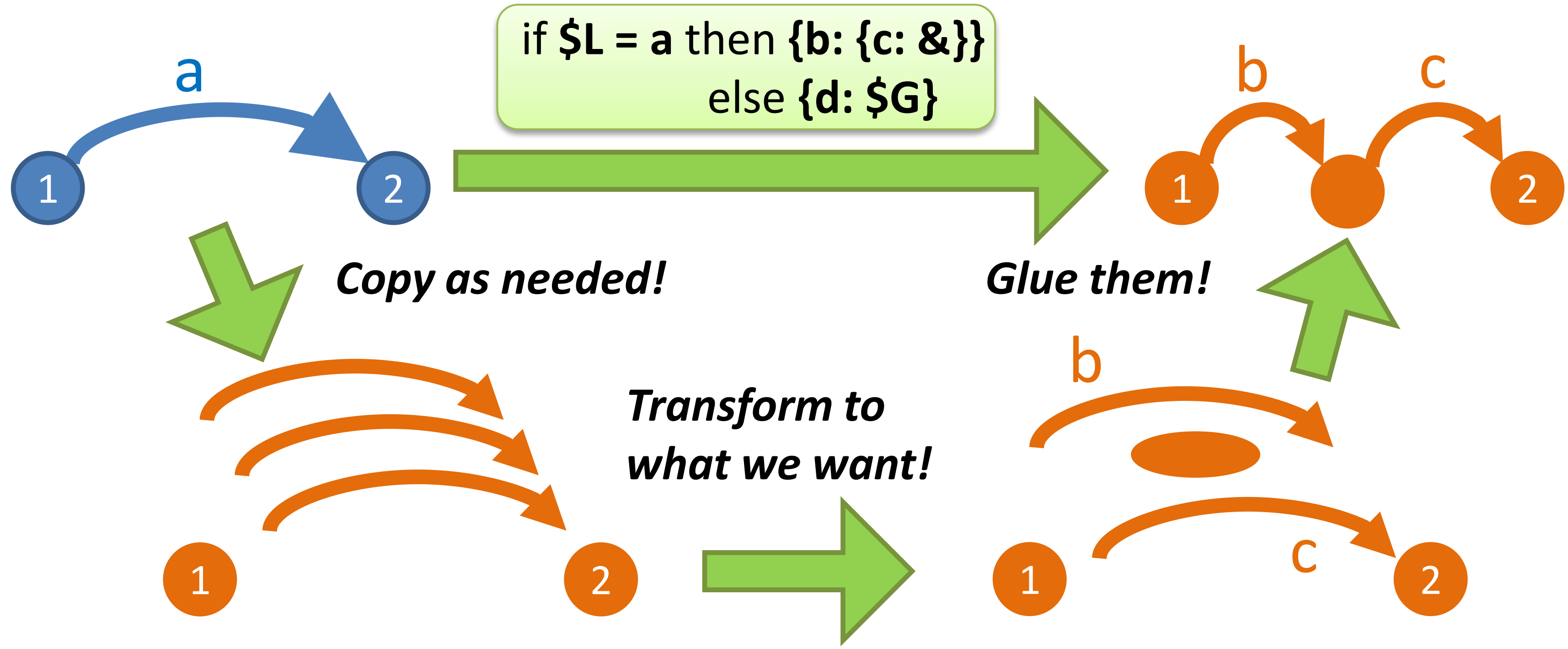
```
E ::= {L:E, L:E, …, L:E}
    | if L=L then E else E
    | $G
    | &
    | rec(λ($L,$G). E)(E)
L ::= (label constant)
     | $L
```

“Bulk” Semantics of UnCAL

```
rec(λ($L,$G).
  if $L = a then {b: {c: &}}
            else {d: $G}
)($input_db)
```

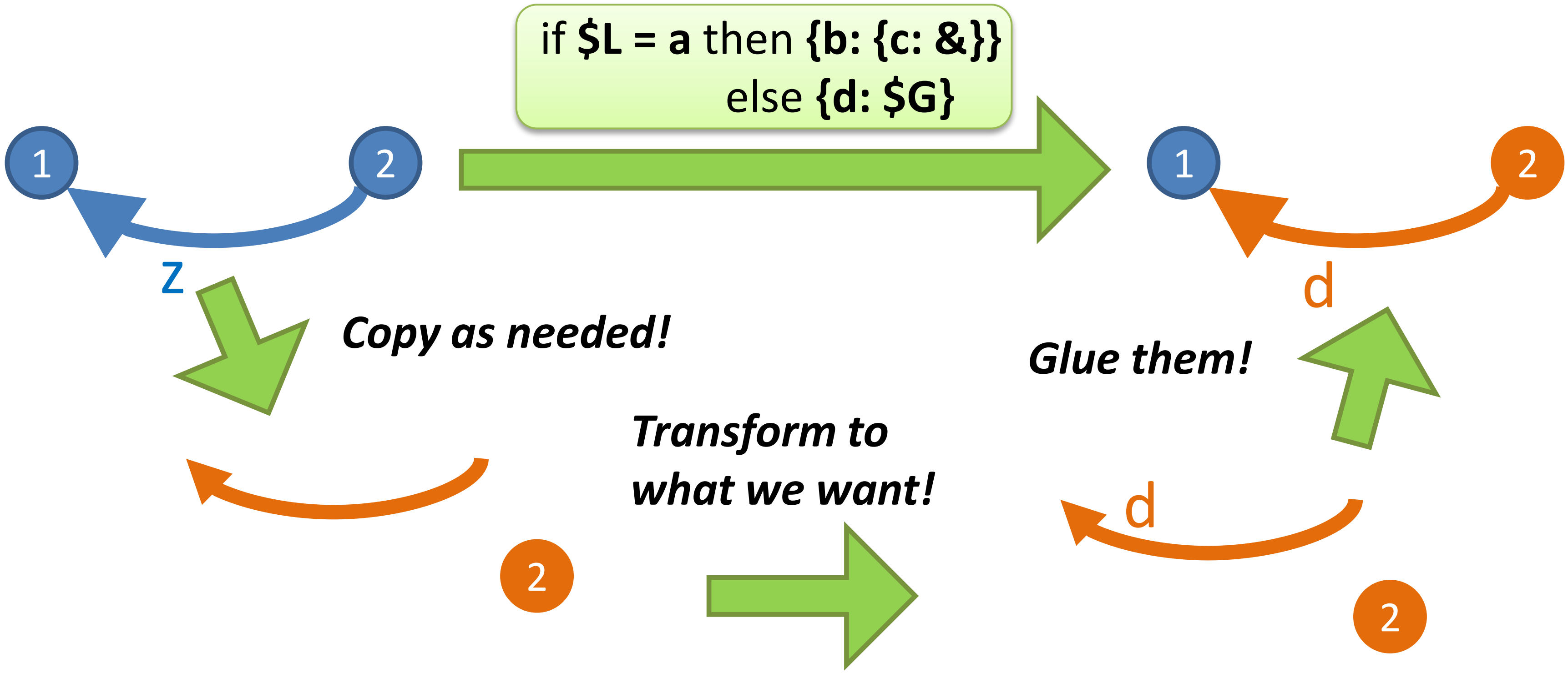


More Precise, MSO-Representable "Finite-Copy" Semantics



$$\mathrm{edge}_b(v,e,u) \Leftrightarrow$$
$$\quad \exists v'\ e'\ u'.\ \mathrm{edge}_a(v',e',u')\ \&\ v=v'_1\ \&\ e=e'_1\ \&\ u=e'_2$$
$$\mathrm{edge}_c(v,e,u) \Leftrightarrow$$
$$\quad \exists v'\ e'\ u'.\ \mathrm{edge}_a(v',e',u')\ \&\ v=e'_2\ \&\ e=e'_3\ \&\ e=u'_1$$

“Finite-Copy” Semantics



$$\mathrm{edge}_d(v,e,u) \Leftrightarrow$$
$$\exists v'\ e'\ u'.\ \neg\mathrm{edge}_a(v',e',u')\ \&\ v=v'_1\ \&\ e=e'_1\ \&\ u=u'$$

Transformation to MSO

Theorem:

Nest-free UnCAL is representable by finite-copying MSO transduction.

```
rec(λ($L,$G).
  if $L = a
    then {b: {c: &}}
    else {d: $G}
)($input_db)
```

$edge_b(v,e,u) \Leftrightarrow edge_a(v',e',u')\ \&\ v=v'_1\ \&\ e=e'_1\ \&\ u=e'_2$

$edge_c(v,e,u) \Leftrightarrow edge_a(v',e',u')\ \&\ v=e'_2\ \&\ e=e'_3\ \&\ e=u'_1$

$edge_d(v,e,u) \Leftrightarrow \neg edge_a(v',e',u')\ \&\ v=v'_1\ \&\ e=e'_1\ \&\ u=u'$

((Transformation = Definition of the output-graph in terms of the input graph))

"Backward" Inference [Courcelle 1994]

MSO Formula stating
"output conforms to the schema"
in terminology of OUTPUT GRAPHS

class OUTPUT { reference result*: MEM; }
class MEM { reference friend*: MEM;
reference name: STRI

$\exists^{set}$OUTPUT. $\exists^{set}$MEM.
(∀x. start(x) → x∈OUTPUT)
∧ (∀x∈OUTPUT. ∀e. ∀u.
edge(x,e,u) → $\text{edge}_{\text{result}}$(x,e,u) & u∈MEM) ∧ ...

OUTPUT GRAPH description
by the INPUT GRAPH

rec(λ($L,$G).
if **$L = a** then **{b: {c: &}}** else **{d: $G}**
)($input_db)

$\text{edge}_b(v,e,u) \Leftrightarrow \text{edge}_a(v',e',u')\ \&\ v=v'_1\ \&\ e=e'_1\ \&\ u=e'_2$
$\text{edge}_c(v,e,u) \Leftrightarrow \text{edge}_a(v',e',u')\ \&\ v=e'_2\ \&\ e=e'_3\ \&\ e=u'_1$
$\text{edge}_d(v,e,u) \Leftrightarrow \neg\text{edge}_a(v',e',u')\ \&\ v=v'_1\ \&\ e=e'_1\ \&\ u=u'$

MSO Formula stating
"output conforms to the schema"
in terminology of INPUT GRAPHS: edge(v',e',u')

Verify this is valid for any INPUT GRAPHS!!

Note: Harder Case

- Nested Recursion (arising from "cross product")

  cannot be encoded into finite-copy semantics

```
select {p: {f: $G1, s:$G2}}
  where {_: $G1} in $db,
        {_ : $G2} in $db
```

```
rec(λ($L1,$G1).
  rec(λ($L2,$G2).
    {pair: {first: $G1, second: $G2}}
  )($db)
)($db)
```

Currently we ask programmer to add annotation →

```
rec(λ($L1,$G1).  rec(λ($L2,$G2).
    {pair: {first: ($G1 :: MEM),
            second: $G2}} ...
```

(why we need the power of MSO?)

- E.g., for "regular path pattern"

```
select {result: $x}
where
  { _*: $x},
  {name:  John} in $x
```

```
select {result: $x}
where
  { (a|b).(c|d)*: $x},
  {name:  John} in $x
```

- MSO can encode finite automata

$\exists^{set}$ Q_1. $\exists^{set}$ Q_2. … $\exists^{set}$ Q_n. "there is a run of the automaton that reaches states Q's on each node"

MSO Validation

Now we have a **MSO Formula on Graphs**.

?

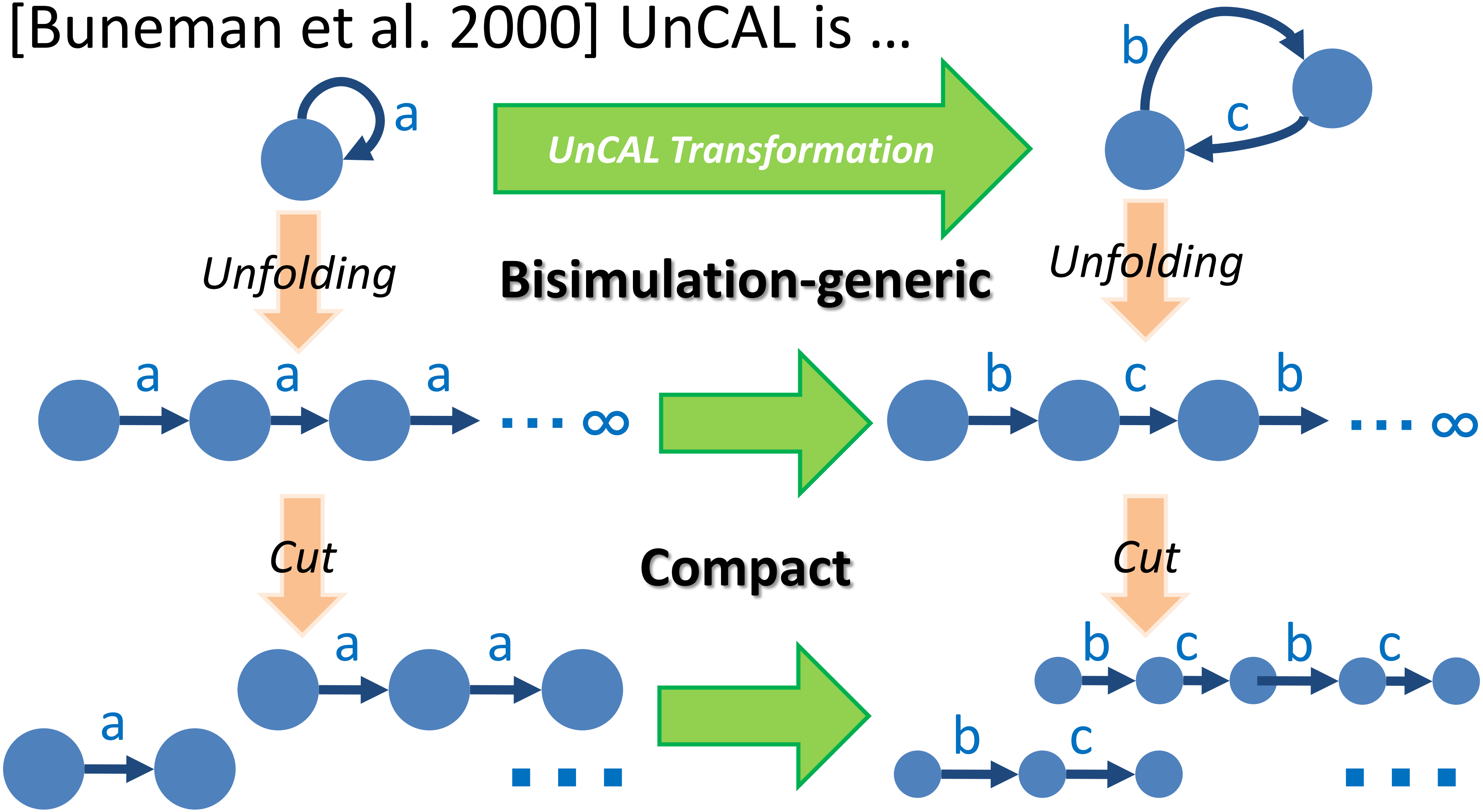
MSO (even 1st-Order Logic) on Graphs is undecidable [Trakhtenbrot 1950].

MONA [Henriksen et al., 1995]

can decide validness of **MSO on Finite Trees**.

Two Nice Props of UnCAL

[Buneman et al. 2000] UnCAL is …



MSO Validation

Now we have a **MSO Formula on Graphs**.

MSO (even 1st-Order Logic) on Graphs is undecidable [Trakhtenbrot 1950].

!

Theorem: If MSO formula is Bisimulation-Generic and Compact, it is valid on graphs iff on finite trees.

MONA [Henriksen et al., 1995]

can decide validness of **MSO on Finite Trees**.

Discussion: Choice of Logic

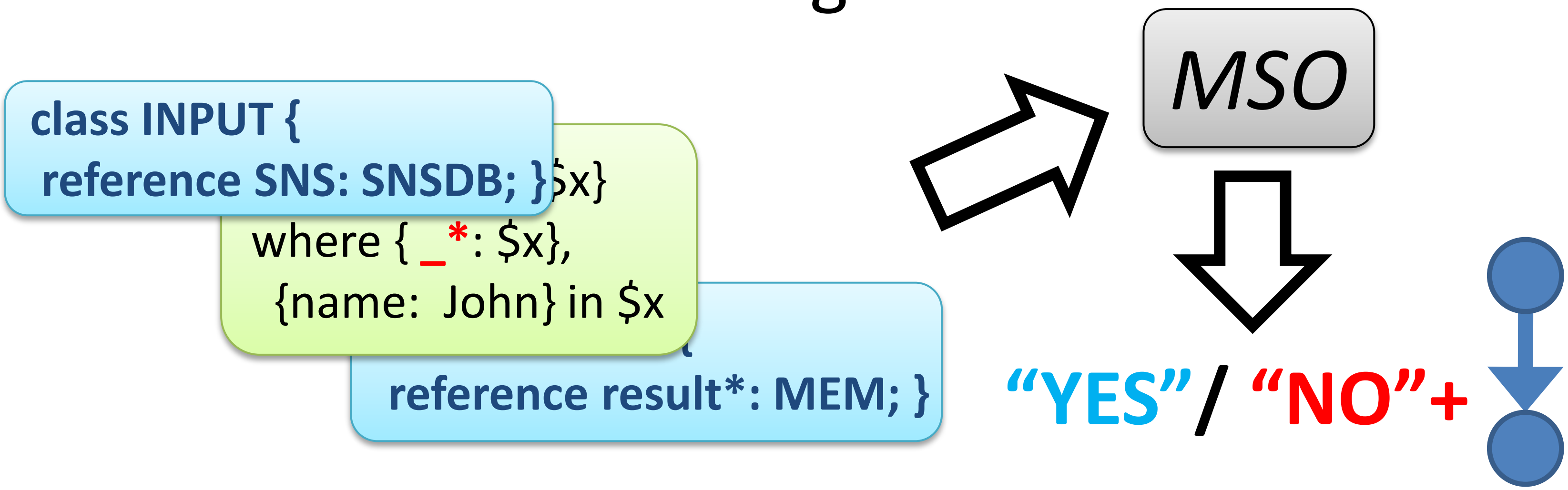
- **MSO**
  - **Powerful, yet decidable (if we fully utilize bisimulation)**
- FO+TC (1st-Order Logic + Transitive Closure)
  - Very powerful; express all UnCAL without annotation
  - Undecidable, even on finite trees
- FO (1st-Order), SMT (Satisfiability Modulo Theory)
  - Very good solvers
  - Too weak for schemas or UnCAL; cannot use repetition
- mu-Calculus (Modal Logic with Fixpoint)
  - Theoretically, equal to MSO under bisimulation
  - Not clear how to represent transformations

Discussion: Approach

- **Our Approach**
  - **"Backward" (define the output by the input, with logic)**
- Other Possible Approaches
  - "Forward" (e.g., abstract interpretation)
    - [Buneman et al., 1997] [Nakano, **Today!**]
    - Better in range analysis. Worse in counterexample generation.
  - "Type System"
    - Hard, because we need context-dependent types, etc.
    - Two $G may differ in types: if $L=a then …$G… else …$G…

Summary

Verify "type-correctness" of graph transformations via Monadic 2nd-Order Logic



- **<u>MSO</u>** and **<u>bisimulation</u>** are good tools for graphs!
- Future work : checking other properties